Higher-Order Euclidean Sets

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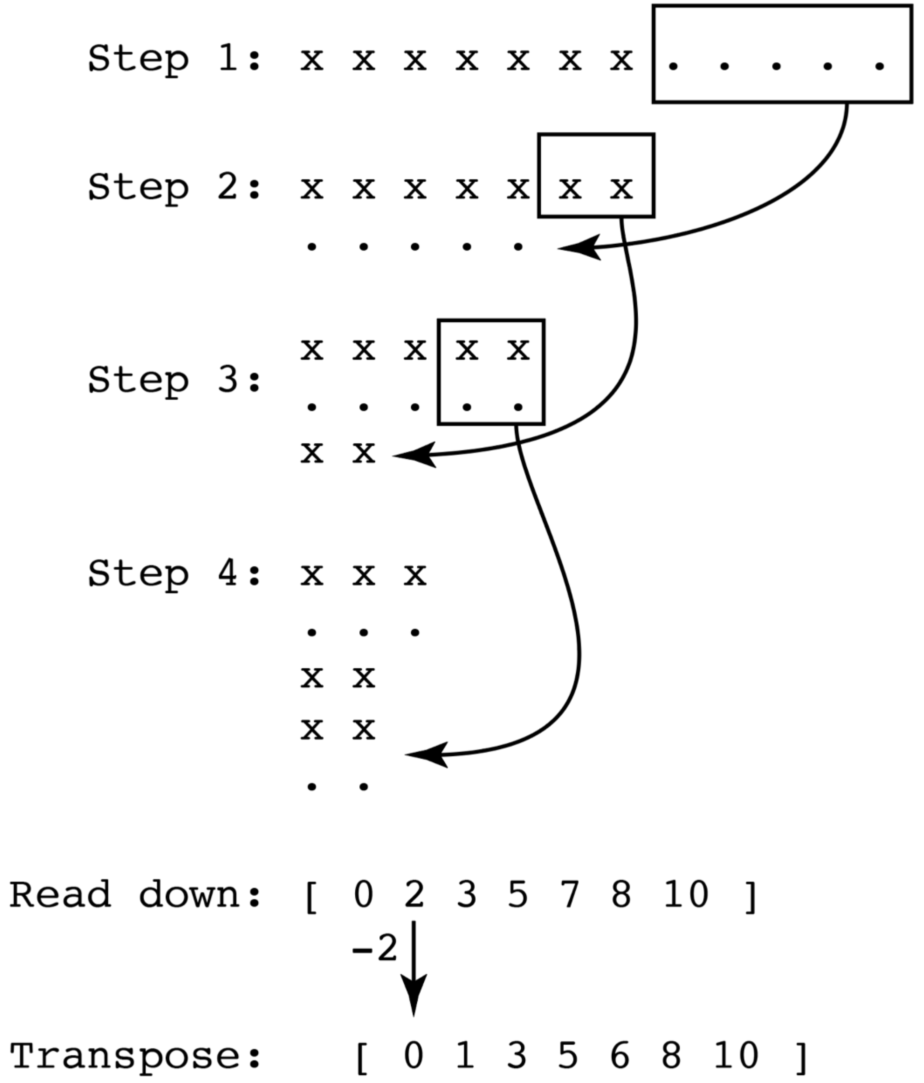
Example 1: E(7, 12)

Clough and Douthett 1991
Method for computing a maximally even set

$$E_\alpha(k_1, n_1) = \left\{ \left\lfloor \frac{in_1}{k_1} \right\rfloor : i = 0, 1, \ldots, k_1 - 1 \right\}$$

n	k	i	calculate	round down
12	7	0	0	0
12	7	1	1.71428571	1
12	7	2	3.42857143	3
12	7	3	5.14285714	5
12	7	4	6.85714286	6
12	7	5	8.57142857	8
12	7	6	10.2857143	10

Toussaint 2005
Geometric method for computing a Euclidean set

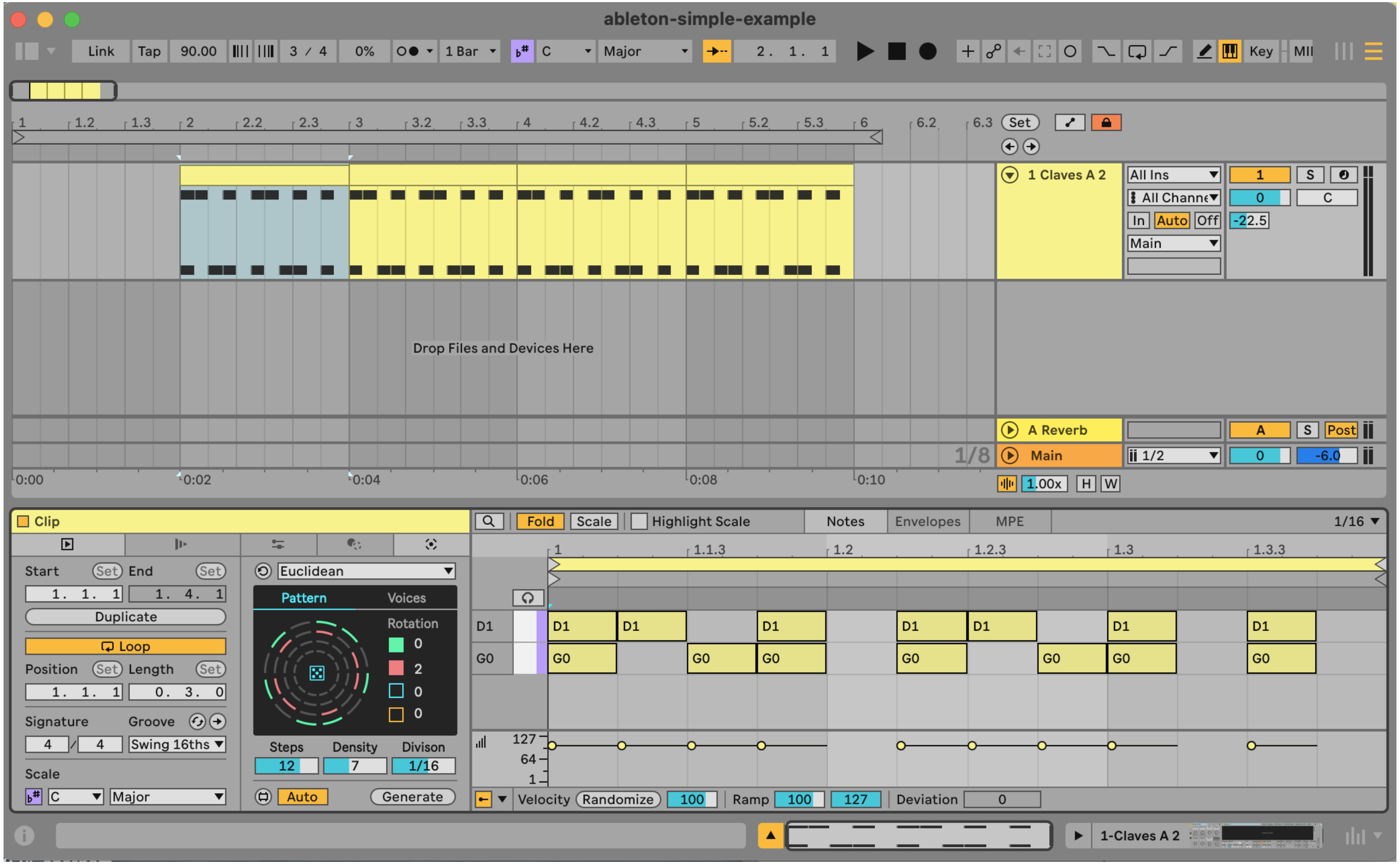


Example 2: E(7,12): Ableton Live

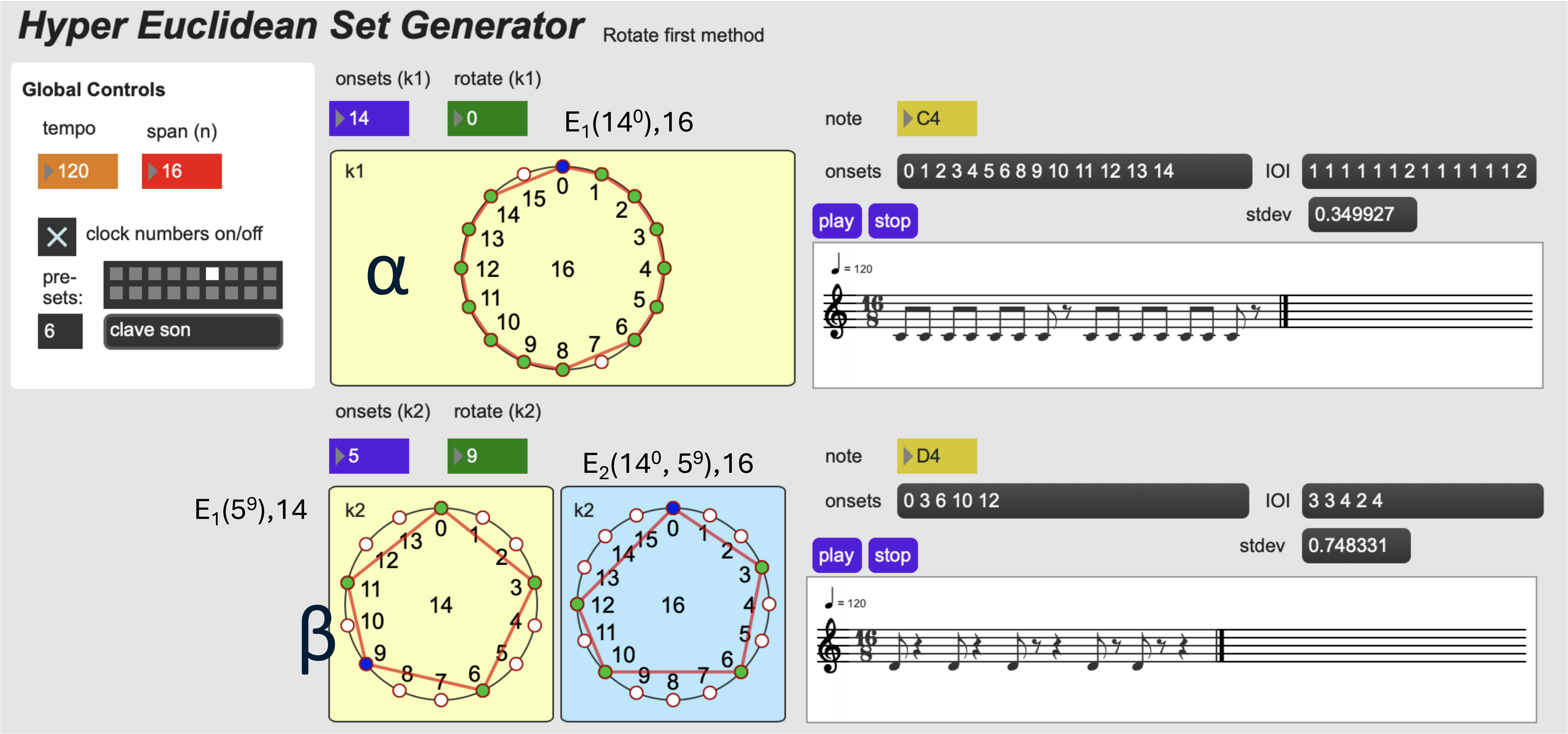
Top part:
[0 1 3 5 6 8 10]

Bottom part:
[0 2 3 5 7 8 10]

These two cycles are the same underlying structure, displaced by 2 time points.



Example 3: Clave – rotate first algorithm



Example 4: Clave – rotate first algorithm

266: HE (16 14^0 5^9) = [0 3 6 10 12] < 3 3 4 2 4 >

394: HE (16 14^8 5^9) = [0 3 6 10 12] < 3 3 4 2 4 >

569: HE (16 13^3 5^8) = [0 3 6 10 12] < 3 3 4 2 4 >

649: HE (16 13^8 5^8) = [0 3 6 10 12] < 3 3 4 2 4 >

676: HE (16 13^10 5^3) = [0 3 6 10 12] < 3 3 4 2 4 >

756: HE (16 13^15 5^3) = [0 3 6 10 12] < 3 3 4 2 4 >

801: HE (16 12^2 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

813: HE (16 12^2 5^12) = [0 3 6 10 12] < 3 3 4 2 4 >

865: HE (16 12^6 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

877: HE (16 12^6 5^12) = [0 3 6 10 12] < 3 3 4 2 4 >

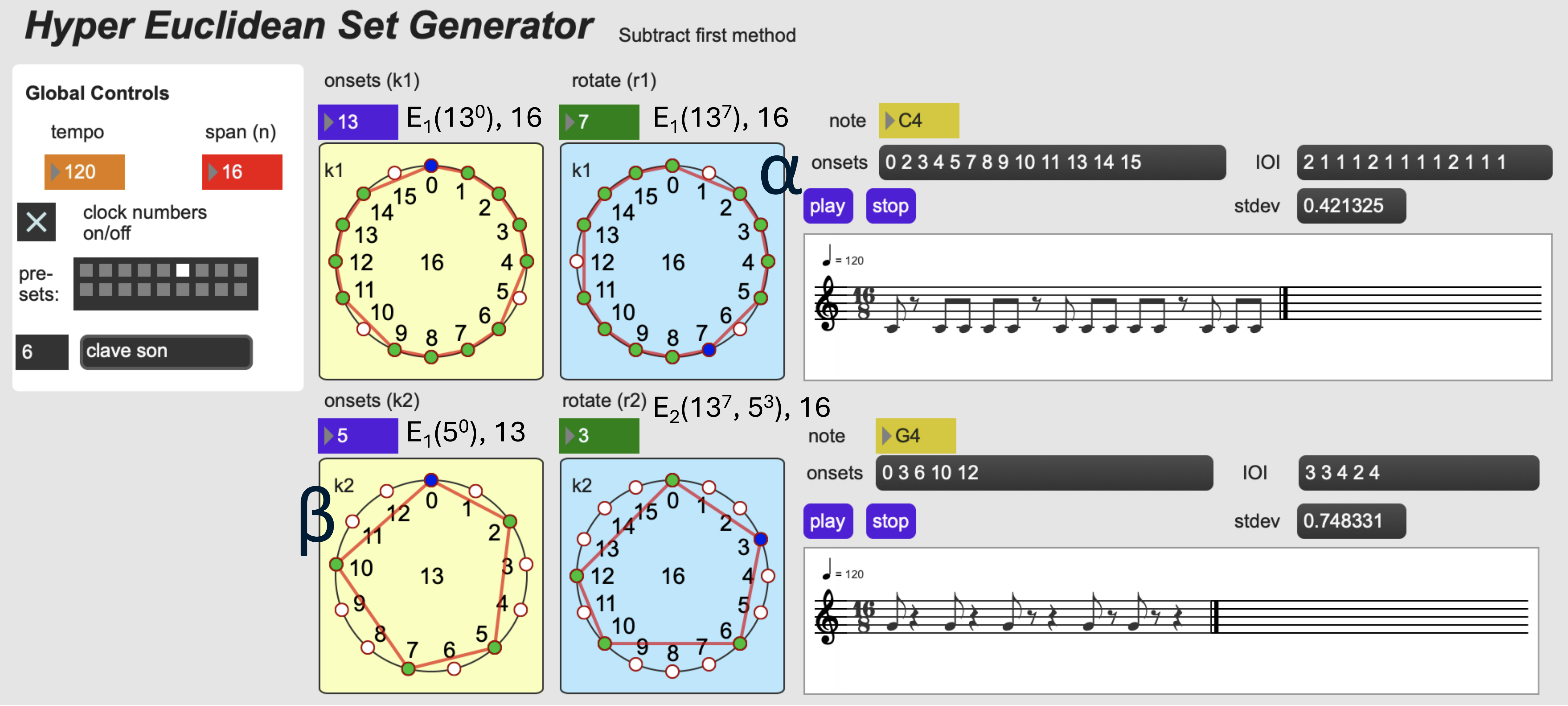
929: HE (16 12^10 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

941: HE (16 12^10 5^12) = [0 3 6 10 12] < 3 3 4 2 4 >

993: HE (16 12^14 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

1005: HE (16 12^14 5^12) = [0 3 6 10 12] < 3 3 4 2 4 >

Example 5: Clave – subtract first algorithm



Example 6: Clave – subtract first algorithm

363: HE (16 14^6 5^10) = [0 3 6 10 12] < 3 3 4 2 4 >

491: HE (16 14^14 5^10) = [0 3 6 10 12] < 3 3 4 2 4 >

628: HE (16 13^7 5^3) = [0 3 6 10 12] < 3 3 4 2 4 >

667: HE (16 13^9 5^10) = [0 3 6 10 12] < 3 3 4 2 4 >

708: HE (16 13^12 5^3) = [0 3 6 10 12] < 3 3 4 2 4 >

747: HE (16 13^14 5^10) = [0 3 6 10 12] < 3 3 4 2 4 >

801: HE (16 12^2 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

865: HE (16 12^6 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

929: HE (16 12^10 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

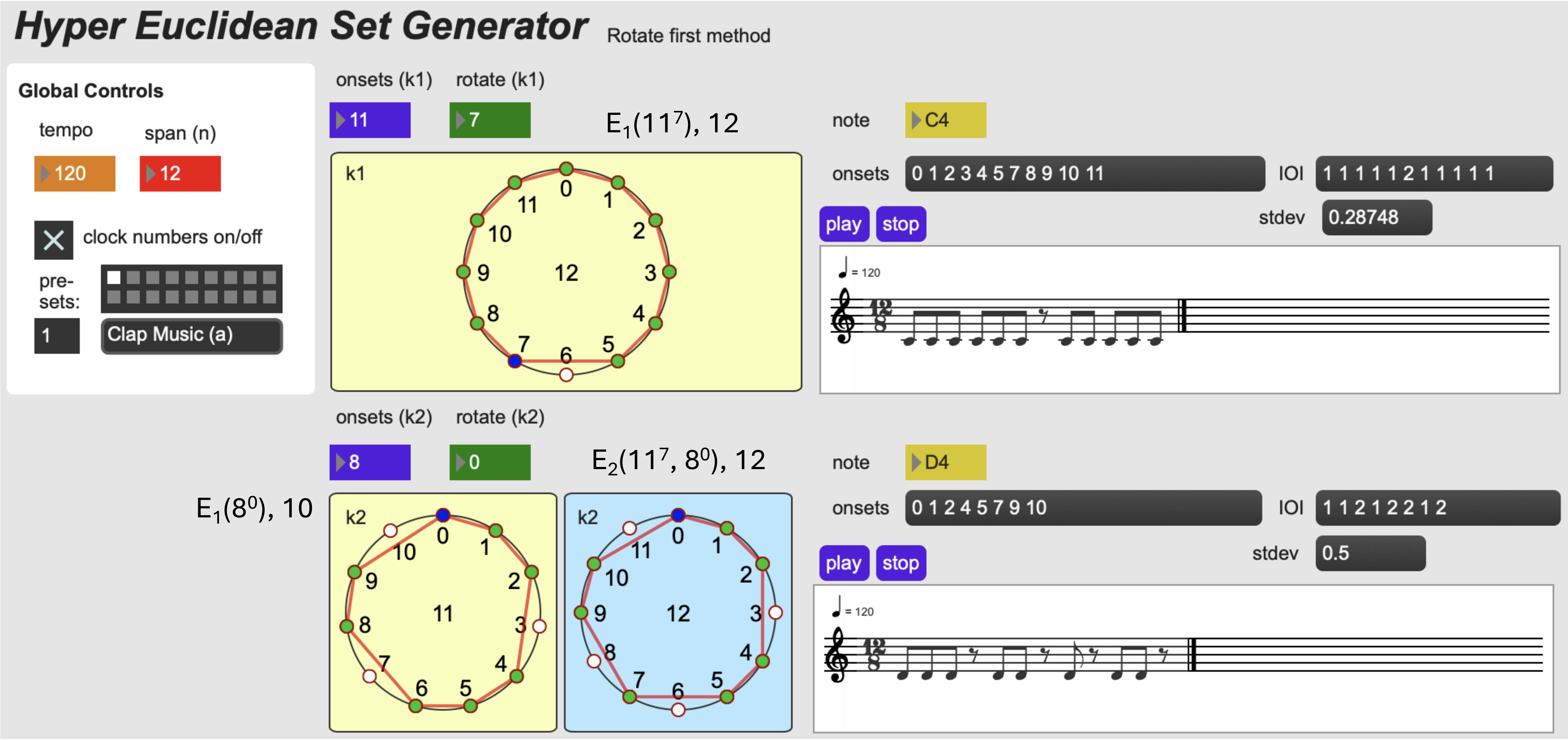
993: HE (16 12^14 5^0) = [0 3 6 10 12] < 3 3 4 2 4 >

Example 7. A "*Slate*" for clave timeline calculation of depth 2 (p = 2)

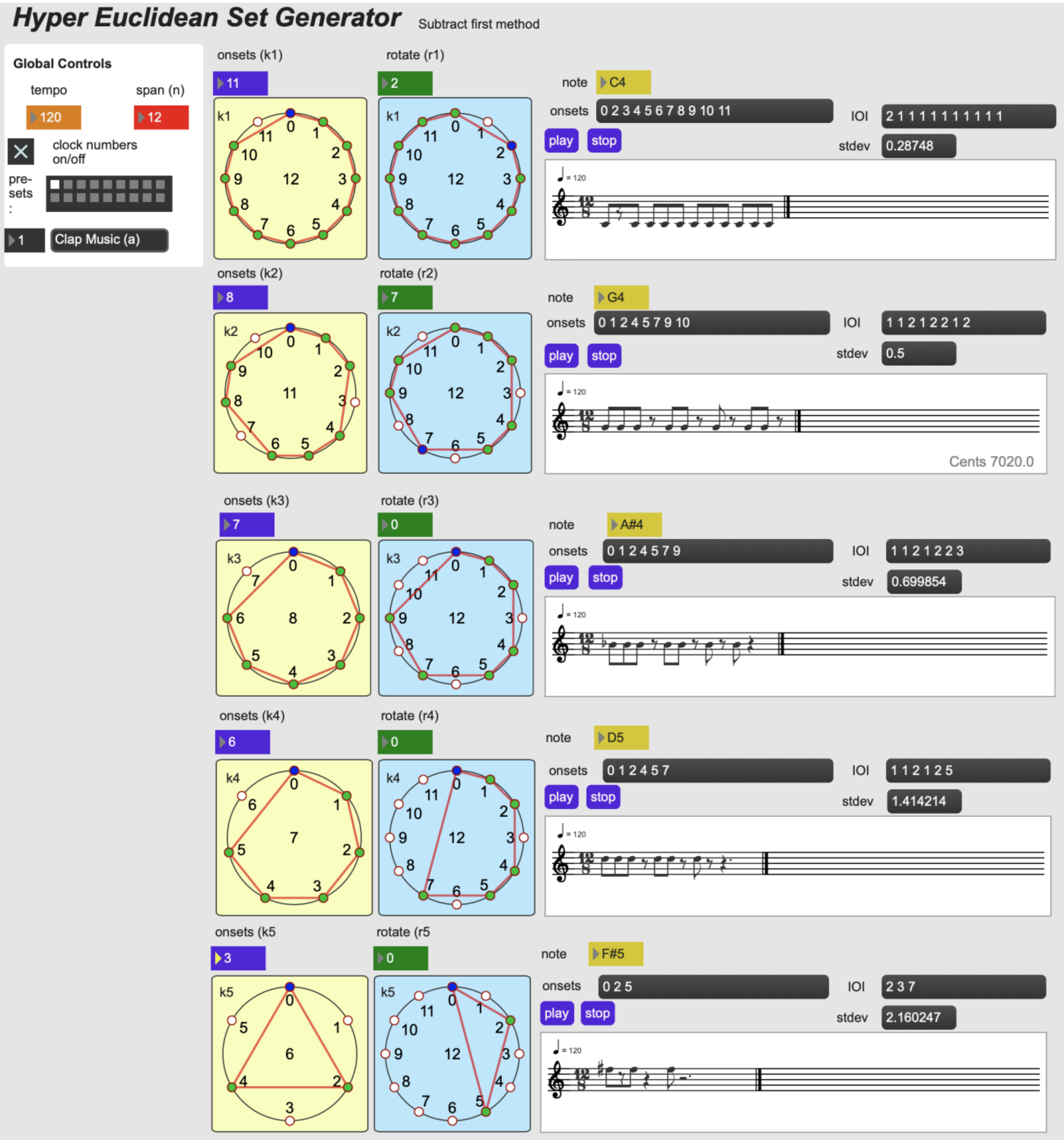
	n	k_1	k_2	number of sets
Row 0:	16	15	5	16x16 = 256
Row 1:	16	14	5	256
Row 2:	16	13	5	256
Row 3:	16	12	5	256
Row 4:	16	11	5	256
Row 5:	16	10	5	256
Row 6:	16	9	5	256
Row 7:	16	8	5	256
Row 8:	16	7	5	256
Row 9:	16	6	5	256
				====
				sum: 2560

Unfortunately, complexity increases at $O(n^p)$.

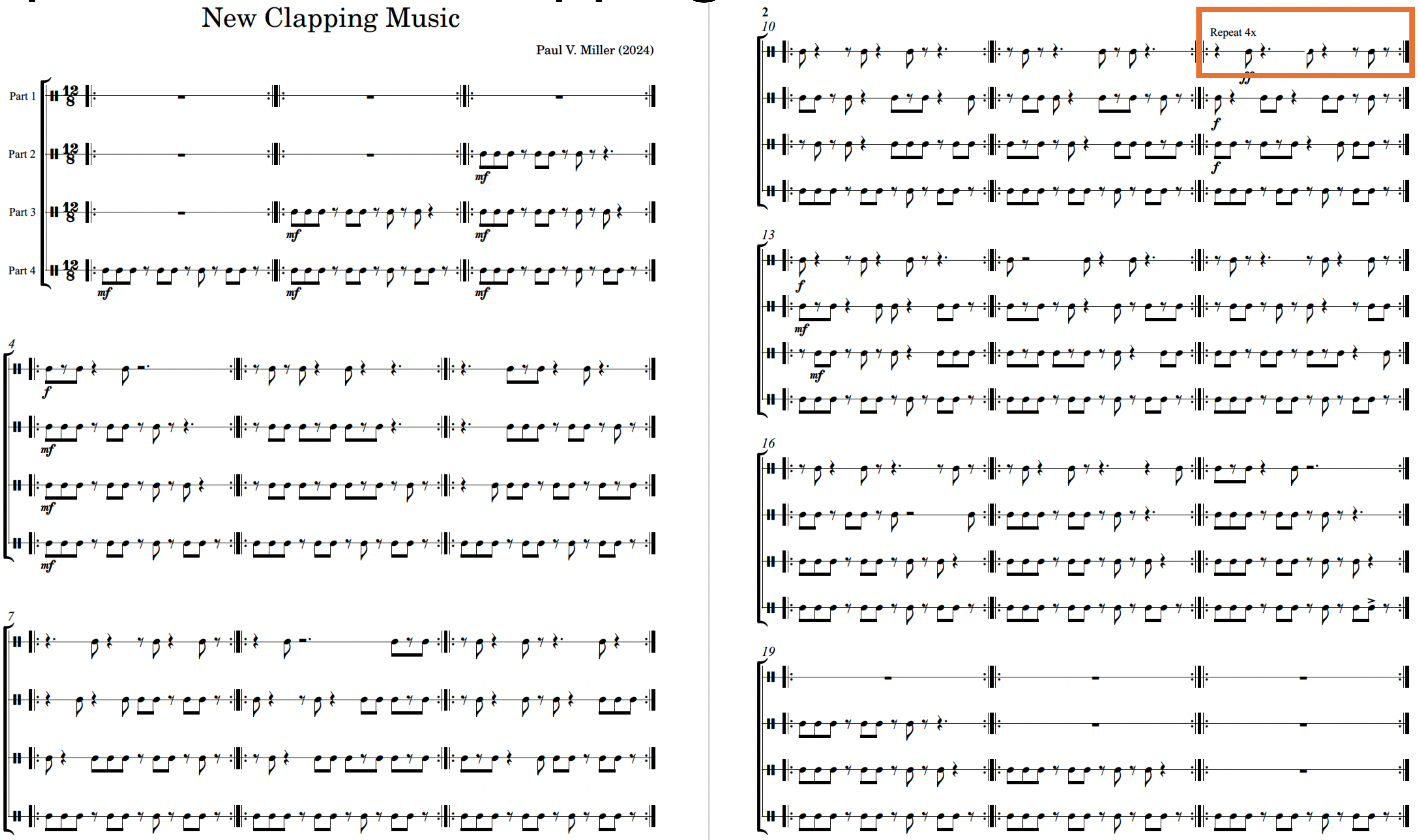
Example 8. "Clapping Music" timeline



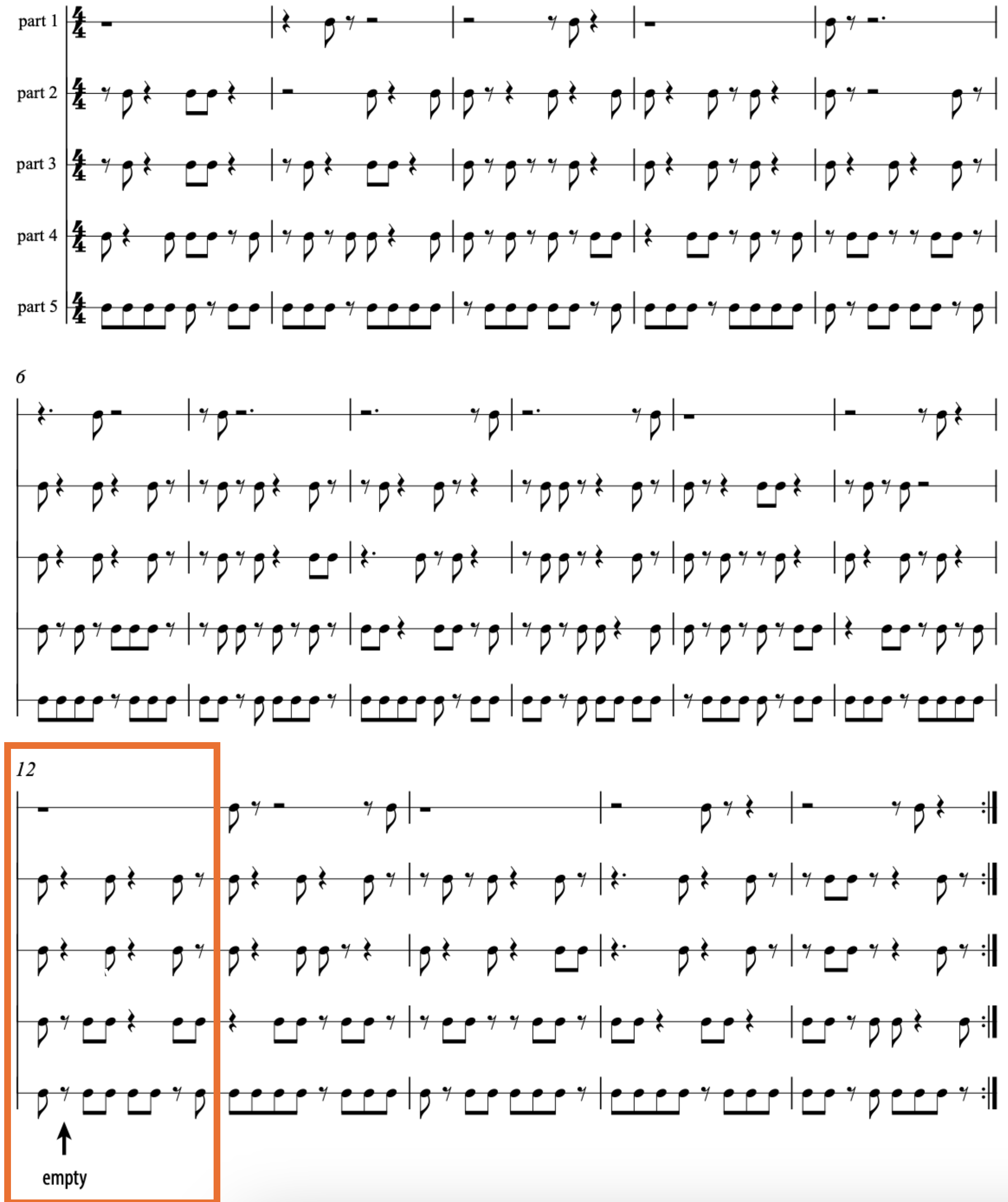
Example 9. Deriving a fifth-order Euclidean set from the Clapping Music timeline



Example 10. “New Clapping Music”



Example 11. An example of a composition using higher-order Euclidean rhythms (105^{0}, 71^{3}, 48^{9}, 43^{11}, 12^{20}), 128



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Appendix 1: Producing higher-order sets

The maximally-even algorithm (Clough and Douthett 1991) produces a conventional first-order Euclidean set E_α where n is the set's span, and k represents the number of desired onsets:

$$E_\alpha(k_1, n_1) = \left\{ \left\lfloor \frac{in_1}{k_1} \right\rfloor : i = 0, 1, ..., k_1 - 1 \right\}$$

E_α is a set of onsets where j is the index of each e:

$$e_{\alpha_j} \in E_\alpha : j = 1, 2, ..., k_1$$

To produce a **second-order Euclidean set** E_2, let $n_2 = k_1$, and k_2 equal the number of desired onsets in the second iteration. Then by simple recursion,

$$E_\beta(k_2, n_2) = \left\{ \left\lfloor \frac{in_2}{k_2} \right\rfloor : i = 0, 1, ..., k_2 - 1 \right\}$$

produces an **index set** such that each $e_{\beta_j} \in E_\beta$ is the jth term in E_α. The set of e_β onsets in E_α is the second-order Euclidean set E_2. Consequently, E_2 is *not* necessarily a subset of E_α.

Appendix 2. Deriving higher-order Euclidean sets for the example composition, *n*=128.

Euclidean set (128, 105^0) is: [0 1 2 3 4 6 7 8 9 10 12 13 14 15 17 18 19 20 21 23 24 25 26 28 29 30 31 32 34 35 36 37 39 40 41 42 43 45 46 47 48 49 51 52 53 54 56 57 58 59 60 62 63 64 65 67 68 69 70 71 73 74 75 76 78 79 80 81 82 84 85 86 87 88 90 91 92 93 95 96 97 98 99 101 102 103 104 106 107 108 109 110 112 113 114 115 117 118 119 120 121 123 124 125 126]
IOI is: < 1 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 2 >

Hyper Euclidean set (128 105^0 71^3) = [0 3 4 5 7 9 11 12 15 16 18 20 22 23 26 27 29 31 33 34 37 38 40 42 44 45 46 49 50 52 54 56 57 60 61 63 65 67 68 71 72 74 76 78 79 82 83 85 87 88 90 91 94 95 98 99 101 102 105 106 109 110 112 113 116 117 120 121 123 124 127]
HyperEuclid IOI is: < 3 1 1 2 2 2 1 3 1 2 2 2 1 3 1 2 2 2 1 3 1 2 2 2 1 1 3 1 2 2 2 1 3 1 2 2 2 1 3 1 2 2 2 1 3 1 2 2 1 2 1 3 1 3 1 2 1 3 1 3 1 2 1 3 1 3 1 2 1 3 1 >

Hyper Euclidean set (105^0 71^3 48^9) = [1 4 5 9 12 13 16 18 21 24 27 29 32 35 38 40 43 46 49 51 54 55 59 61 65 66 70 72 74 77 80 83 85 88 91 94 96 99 100 104 107 110 111 115 118 121 122 126]
HyperEuclid IOI is: < 3 1 4 3 1 3 2 3 3 3 2 3 3 3 2 3 3 3 2 3 1 4 2 4 1 4 2 2 3 3 3 2 3 3 3 2 3 1 4 3 3 1 4 3 3 1 4 3 >

Hyper Euclidean set (128 105^0 71^3 48^9 43^11) = [1 4 5 12 15 16 20 23 24 27 29 32 38 40 43 46 49 51 54 57 60 65 66 70 72 76 77 81 83 88 91 94 96 99 102 105 107 110 115 118 121 122 126]
HyperEuclid IOI is: < 3 1 7 3 1 4 3 1 3 2 3 6 2 3 3 3 2 3 3 3 5 1 4 2 4 1 4 2 5 3 3 2 3 3 3 2 3 5 3 3 1 4 3 >

Hyper Euclidean set (128 105^0 71^3 48^9 43^11 12^20) = [10 21 32 43 49 63 71 85 96 103 116 125]
HyperEuclid IOI is: < 11 11 11 6 14 8 14 11 7 13 9 13 >

Appendix 3. The harmonic minor scale as a second-order Euclidean set